## Exercise 22B: Solutions

**1 a**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $\Pr(X = x)$ | 0.16 | 0.48 | 0.36 |

**b** $\Pr(X \geq 1) = 0.36 + 0.48$
$= 0.84$

**2 a** $\Pr(X = 3) = 0.35$

**b** $\Pr(X < 3) = \Pr(X = 1) + \Pr(X = 2)$
$= 0.05 + 0.15$
$= 0.20$

**c** $\Pr(X \geq 4) = \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6)$
$= 0.25 + 0.15 + 0.05$
$= 0.45$

**d** $\Pr(1 < X < 5) = \Pr(X = 2) + \Pr(X = 3) + \Pr(X = 4)$
$= 0.75$

**e** $\Pr(X \neq 5) = 1 - \Pr(X = 5)$
$= 1 - 0.15$
$= 0.85$

**f** $\Pr(1 < X < 5 \mid X > 1) = \dfrac{0.75}{0.95} = \dfrac{15}{19}$

**3 a** $\Pr(\hat{P} = 0.2) = 0.0034$

**b** $\Pr(\hat{P} < 0.4) = \Pr(\hat{P} = 0) + \Pr(\hat{P} = 0.2)$
$= 0.0001 + 0.0034$
$= 0.0035$

**c** $\Pr(\hat{P} \geq 0.8) = \Pr(\hat{P} = 0.8) + \Pr(\hat{P} = 1)$
$= 0.4372 + 0.3060$
$= 0.7342$

**d** $\Pr(0.2 < \hat{P} < 0.8) = \Pr(\hat{P} = 0.4) + \Pr(\hat{P} = 0.6)$
$= 0.0422 + 0.2111 + 0.3060$
$= 0.2533$

**e** $\Pr(\hat{P} < 0.8 \mid \hat{P} > 0) = \dfrac{\Pr(0 < \hat{P} < 0.8)}{\Pr(\hat{P} > 0)}$
$= \dfrac{0.2111}{0.9999}$
$= 0.2212$

**f** $\Pr(0.2 < \hat{P} < 0.8 \mid \hat{P} > 0.4) = \dfrac{\Pr(0.2 < \hat{P} < 0.8)}{\Pr(\hat{P} > 0.4)}$
$= \dfrac{0.2533}{0.2111 + 0.4372 + 0.3060}$
$= 0.2654$

**4 a** $p = \dfrac{8}{16} = 0.5$

**b** Number of soft centred chocolates could be $0, 1, 2$ or $3$. Thus, possible values of $\hat{P}$ are $0, \dfrac{1}{3}, \dfrac{2}{3}, 1$

**c** $\Pr(\hat{P}=0) = \Pr(X=0)$

$$= \frac{\binom{8}{0}\binom{8}{3}}{\binom{16}{3}}$$

$$= \frac{56}{560}$$

$$= 0.1$$

$\Pr(\hat{P}=\frac{1}{3}) = \Pr(X=1)$

$$= \frac{\binom{8}{1}\binom{8}{2}}{\binom{16}{3}}$$

$$= \frac{224}{560}$$

$$= 0.4$$

$\Pr(\hat{P}=\frac{2}{3}) = \Pr(X=2)$

$$= \frac{\binom{8}{2}\binom{8}{1}}{\binom{16}{3}}$$

$$= \frac{224}{560}$$

$$= 0.4$$

$\Pr(\hat{P}=1) = \Pr(X=3)$

$$= \frac{\binom{8}{3}\binom{8}{0}}{\binom{16}{3}}$$

$$= \frac{56}{560}$$

$$= 0.1$$

| $\hat{p}$ | $0$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $1$ |
|---|---|---|---|---|
| $\Pr(\hat{P}=\hat{p})$ | $0.1$ | $0.4$ | $0.4$ | $0.1$ |

**d** $\Pr(\hat{P} > 0.25) = 0.9$

**5 a** $p = \frac{12}{20} = 0.6$

**b** Number of male swimmers could be $0, 1, 2, 3, 4$
The values of $\hat{P}$ are $0, 0.2, 0.4, 0.6, 0.8, 1$

**c**

$\Pr(\hat{P}=0) = \Pr(X=0)$

$$= \frac{\binom{12}{0}\binom{8}{5}}{\binom{20}{5}}$$

$$= 0.0036$$

$\Pr(\hat{P}=\frac{1}{5}) = \Pr(X=1)$

$$= \frac{\binom{12}{1}\binom{8}{4}}{\binom{20}{5}}$$

$$= 0.0545$$

$\Pr(\hat{P}=\frac{2}{5}) = \Pr(X=2)$

$$= \frac{\binom{12}{2}\binom{8}{3}}{\binom{16}{3}}$$

$$= 0.2384$$

$$\begin{aligned}\Pr(\hat{P}=\tfrac{3}{5}) &= \Pr(X=3)\\ &= \frac{\binom{12}{3}\binom{8}{2}}{\binom{20}{5}}\\ &= 0.3973\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P}=\tfrac{4}{5}) &= \Pr(X=4)\\ &= \frac{\binom{12}{4}\binom{8}{1}}{\binom{20}{5}}\\ &= 0.2554\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P}=1) &= \Pr(X=5)\\ &= \frac{\binom{12}{5}\binom{8}{0}}{\binom{20}{5}}\\ &= 0.0511\end{aligned}$$

| $\hat{p}$ | 0 | 0.2 | 0.4 |
|---|---|---|---|
| $\Pr(\hat{P}=\hat{p})$ | 0.0036 | 0.0542 | 0.2384 |

| $\hat{p}$ | 0.6 | 0.8 | 1 |
|---|---|---|---|
| $\Pr(\hat{P}=\hat{p})$ | 0.3973 | 0.2554 | 0.0511 |

**d** $\Pr(\hat{P}>0.7)=0.3065$

**e** $\Pr(0<\hat{P}<0.8)=0.6899$,

$$\begin{aligned}\Pr(\hat{P}<0.8\,|\,\hat{P}>0) &= \frac{\Pr(0<\hat{P}<0.8)}{\Pr(\hat{P}>0)}\\ &= 0.6924\end{aligned}$$

**6 a** $p=\dfrac{15}{50}=0.3$

**b** Possible number of defectives could be $0$, $1$, $2$, $3$, or $4$. Therefore values of $\hat{P}$ are $0, 0.25, 0.5, 0.75, 1$.

**c**

$$\begin{aligned}\Pr(\hat{P}=0) &= \Pr(X=0)\\ &= \frac{\binom{15}{0}\binom{35}{4}}{\binom{50}{4}}\\ &= 0.2274\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P}=\tfrac{1}{4}) &= \Pr(X=1)\\ &= \frac{\binom{15}{1}\binom{35}{3}}{\binom{50}{4}}\\ &= 0.4263\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P}=\tfrac{2}{4}) &= \Pr(X=2)\\ &= \frac{\binom{15}{2}\binom{35}{2}}{\binom{50}{4}}\\ &= 0.2713\end{aligned}$$

$$\begin{aligned}\Pr(\hat{P}=\tfrac{3}{4}) &= \Pr(X=3)\\ &= \frac{\binom{15}{3}\binom{35}{1}}{\binom{50}{4}}\end{aligned}$$

$$= 0.0691$$

$$\Pr(\hat{P} = 1) = \Pr(X = 4)$$
$$= \frac{\binom{12}{4}\binom{35}{0}}{\binom{50}{4}}$$
$$= 0.0059$$

| $\hat{p}$ | 0 | 0.2 | 0.5 |
|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | 0.2274 | 0.4263 | 0.2713 |

| $\hat{p}$ | 0.75 | 1 |
|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | 0.0691 | 0.0059 |

**d** $\Pr(\hat{P} > 0.5) = 0.075$

**e** $\Pr(0 < \hat{P} < 0.5) = 0.4263$, $\Pr(\hat{P} < 0.5 \mid \hat{P} > 0) = 0.5518$

**7 a** $\Pr(\hat{P} > 0.6) = \Pr(\hat{P} = \frac{2}{3}) + \Pr(\hat{P} = 1)$
$= 0.028$

**b** $\Pr(0 < \hat{P} < 0.6) = \Pr(\hat{P} = \frac{1}{3}) = 0.243$,

$$\Pr(\hat{P} < 0.6 \mid \hat{P} > 0) = \frac{\Pr(0 < \hat{P} < 0.6)}{\Pr(\hat{P} > 0)}$$
$$= 0.897$$

**8 a** $p = 0.5$

**b** Values of $\hat{P}$ are $0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1$

**c** Binomial with $n = 10, p = 0.5$

$$\Pr(\hat{P} = 0) = \Pr(X = 0)$$
$$= \binom{10}{0}(0.5)^0(0.5)^{10}$$
$$= 0.00098$$
$$\Pr(\hat{P} = 0.1) = \Pr(X = 1)$$
$$= \binom{10}{1}(0.5)^1(0.5)^9$$
$$= 0.0098$$

The following values are obtained in the same way

| $\hat{p}$ | 0 | 0.1 | 0.2 | 0.3 |
|---|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | 0.00098 | 0.0098 | 0.0440 | 0.1172 |

| $\hat{p}$ | 0.4 | 0.5 | 0.6 | 0.7 |
|---|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | 0.2051 | 0.2461 | 0.2051 | 0.1172 |

| $\hat{p}$ | 0.8 | 0.9 | 1 |
|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | 0.0440 | 0.0098 | 0.00098 |

**d** $\Pr(\hat{P} > 0.5) = 0.3771$

**9 a** The values that $\hat{P}$ can take are $0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1$

**b** Binomial with $n = 6, p = 0.52$

$$\begin{aligned}\Pr(\hat{P} = 0) &= \Pr(X = 0) \\ &= \binom{6}{0}(0.52)^0(0.48)^6 \\ &= 0.0122\end{aligned}$$

Binomial with $n = 6, p = 0.52$

$$\begin{aligned}\Pr(\hat{P} = \tfrac{1}{6}) &= \Pr(X = 1) \\ &= \binom{6}{1}(0.52)^1(0.48)^5 \\ &= 0.0795\end{aligned}$$

The following values are obtained in the same way.

| $\hat{p}$ | $0$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{2}$ |
|---|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | $0.0122$ | $0.0795$ | $0.2153$ | $0.3110$ |

| $\hat{p}$ | $\frac{2}{3}$ | $\frac{5}{6}$ | $1$ |
|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | $0.2527$ | $0.1095$ | $0.0198$ |

**c** $\Pr(\hat{P} > 0.6) = 0.307$

**d** $$\begin{aligned}\Pr(\hat{P} < 0.3 \,|\, \hat{P} < 0.8) &= \frac{\Pr(\hat{P} < 0.3)}{\Pr(\hat{P} < 0.8)} \\ &= 0.1053\end{aligned}$$

**10a** The values that $\hat{P}$ can take are $0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1$

**b**

Binomial with $n = 8, p = 0.8$

$$\begin{aligned}\Pr(\hat{P} = 0) &= \Pr(X = 0) \\ &= \binom{8}{0}(0.8)^0(0.2)^6 \\ &= 0.000003\end{aligned}$$

Binomial with $n = 8, p = 0.8$

$$\begin{aligned}\Pr(\hat{P} = \tfrac{1}{8}) &= \Pr(X = 1) \\ &= \binom{8}{1}(0.8)^1(0.2)^5 \\ &= 0.00008\end{aligned}$$

The following values are obtained in the same way.

| $\hat{p}$ | $0$ | $\frac{1}{8}$ | $\frac{1}{4}$ |
|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | $0.000003$ | $0.00008$ | $0.00115$ |

| $\hat{p}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ |
|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | $0.0092$ | $0.0459$ | $0.1468$ |

| $\hat{p}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | $1$ |
|---|---|---|---|
| $\Pr(\hat{P} = \hat{p})$ | $0.2936$ | $0.3355$ | $0.1678$ |

**c** $\Pr(\hat{P} > 0.6) = 0.1468 + 0.2936 + 0.3355 + 0.1678 = 0.9437$

**d** $$\Pr(\hat{P} > 0.6 \,|\, \hat{P} > 0.25) = \frac{\Pr(\hat{P} > 0.6)}{\Pr(\hat{P} > 0.25)}$$

$$= 0.9448$$

**11a**

| $\hat{p}$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|
| Hyp | 0.0587 | 0.2499 | 0.3827 | 0.2499 | 0.0587 |
| Bin | 0.0625 | 0.25 | 0.375 | 0.25 | 0.0625 |

**b**

| $\hat{p}$ | 0 | 0.1 | 0.2 | 0.3 |
|---|---|---|---|---|
| Hyp | 0.0006 | 0.0072 | 0.0380 | 0.1131 |
| Bin | 0.00098 | 0.0098 | 0.0440 | 0.1172 |

| $\hat{p}$ | 0.4 | 0.5 | 0.6 | 0.7 |
|---|---|---|---|---|
| Hyp | 0.2114 | 0.2593 | 0.2114 | 0.1131 |
| Bin | 0.2051 | 0.2461 | 0.2051 | 0.1172 |

| $\hat{p}$ | 0.8 | 0.9 | 1 |
|---|---|---|---|
| Hyp | 0.0380 | 0.0072 | 0.0006 |
| Bin | 0.0440 | 0.0098 | 0.00098 |

**c** Not much